WHICH GEOID FITS EGYPT BETTER

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Existing precise geoid for any country is important in obtaining precise horizontal and vertical positioning. The trials made to calculate the geoid in Egypt before 1986 covered limited areas and based on one type of the available geodetic data. The first geoid solution covering the whole area of Egypt started in 1986 using heterogeneous data. The second solution was in 1993 and uses also heterogeneous data. In 1995 the geoid in Egypt was computed using gravity data supported by Digital Terrain Model. In 1997, GPS observations with orthometric heights are used to improve the Gravimetric geoid 95 to obtain geoid 97. The main objective of this research is to focus on these four local solutions of the geoid in Egypt and to put them in a comparison among each other and against two of the global geoid solutions, derived from global geopotential models, OSU-81 and OSU-91A. The comparison includes the data used, the techniques followed, and the obtained results. Finally the best solution, among these six geoids will be recommended for use in Egypt.

1. INTRODUCTION

It is well known, and thus an accepted fact, that all geodetic measurements are closely related to the actual gravity field, as being taken on the physical surface of the earth. On the other hand, for geometrical relationships and associated mathematical computations among different terrain points, a mean earth reference ellipsoid is usually chosen to approximate the actual earth. Such ellipsoid generates what is known in practice as the theoretical or normal gravity field. Also, it is practically accepted that the geoid represents the actual gravity field of the earth. The difference between the gravity potential, or the gravity values, between the geoid and mean earth ellipsoid is known as the anomalous gravity field. This anomalous field is the basic parameter for transformations of the geodetic measurements from the physical space of observation (the geoid) to the geometrical space of computations, the ellipsoid [Heiskannen and Moritz, 1967]. This means that a reliable geoid, presented in the form of geoid undulations and deflection of the vertical components, should be available for any country, like Egypt, for precise geodetic and geodynamical applications.

2. THE GEOID IN EGYPT, HISTORICAL BACKGROUND

No one can deny that several trials have been made earlier for geoid determination in Egypt. Such trials were mainly in the form of astro-geodetic profiles along the primary triangulation chains, or geoidal surfaces based on all available geodetic heterogeneous data via a least-squares collocation estimation process. Also, other trials were based on linear interpolations between discrete points, for which the geoid undulations were determined from the combination of geometric Doppler or GPS geometric satellite positioning and leveling operations [Basset and Rogers, 1981; Finnmap, 1989].

On the other hand, the second group of trials, i.e. geoid surfaces, were very useful, but unfortunately encounter inevitable deterioration of accuracy in the predicted geoidal information over areas suffering from lack of gravity and other heterogeneous data. Besides heterogeneous geoids require quite involved and lengthy estimation process [Alnagar, 1986; Nassar et al., 1993]. Moreover, they require different kinds of heterogeneous quantities, having of course different accuracy's, which are inter-correlated through the so-called covariance functions. Such covariance functions involve many assumptions, on both global and regional levels, which amounts to some difficulties concerning their reliability and local applicability. Again in 1993, the geoid in Egypt was computed related to WGS-84 after adding more data using also the collocation technique [Eltokhey, 1993].

El Sagheer [1995], developed a gravimetric geoid for Egypt relative to WGS-84 using FFT technique and for the first time using DTM in the prediction of free-air anomaly in a void areas and in calculating the terrain effect. This time the short, medium and long wave length features of the geoid were taken into account. Shaker, et al. [1997], improve the Egyptian gravimetric geoid 1995 using the available GPS observations in the Eastern Desert, through combining them with the gravimetric undulations. As a result, an enhanced geoid, for the entire Egyptian territory, has been developed.

Based on the above situation, there has been a necessity to look for the best geoid solution. In this context, comparison among each other and against two of the global geoidal solutions derived from OSU-81 and OSU-91A were made. The comparison includes, type of used data, mathematical formulation and obtained results. This is actually the main point of interest in the present paper.

3. THE AVAILABLE GEODETIC DATA IN EGYPT

For the purpose of geoid determination, different kinds of geodetic data can be used, such as, gravity observations, astronomic observations for latitude and longitude, ground positions determined from satellite observations, altimetry data for location of instantaneous sea level, and the harmonic coefficients determined from the analysis of the satellite orbit and terrestrial gravity data. These kinds of heterogeneous data can be used either individually, combined in pairs, or combined all together, and hence define different techniques for geoid determination. Table 1 summarizes the available types of geodetic data in Egypt, as extracted from Nassar, et al., 1993 and El Sagheer, 1995, while the data distribution is given in Figure 1.

Table (1): The Available Geodetic Data in Egypt.

Data type	Classification	Number	std.
	ESA	180	0.4 mgal
Gravity	NGSBN77	77	0.4 mgal
	2 nd	849	1.0 mgal
Astronomic lat.	ESA	146	1.6″
.	MCE	6	2.25″
Astronomic long.	ESA	13	1.6″
	MCE	6	4.0″
	STREG	12	1.5 m
Doppler	ESA	19	1.5 m
	2 nd order	46	2.5 m
GPS	1 St order	31	1.5 m
	2 nd order	358	2.5 m

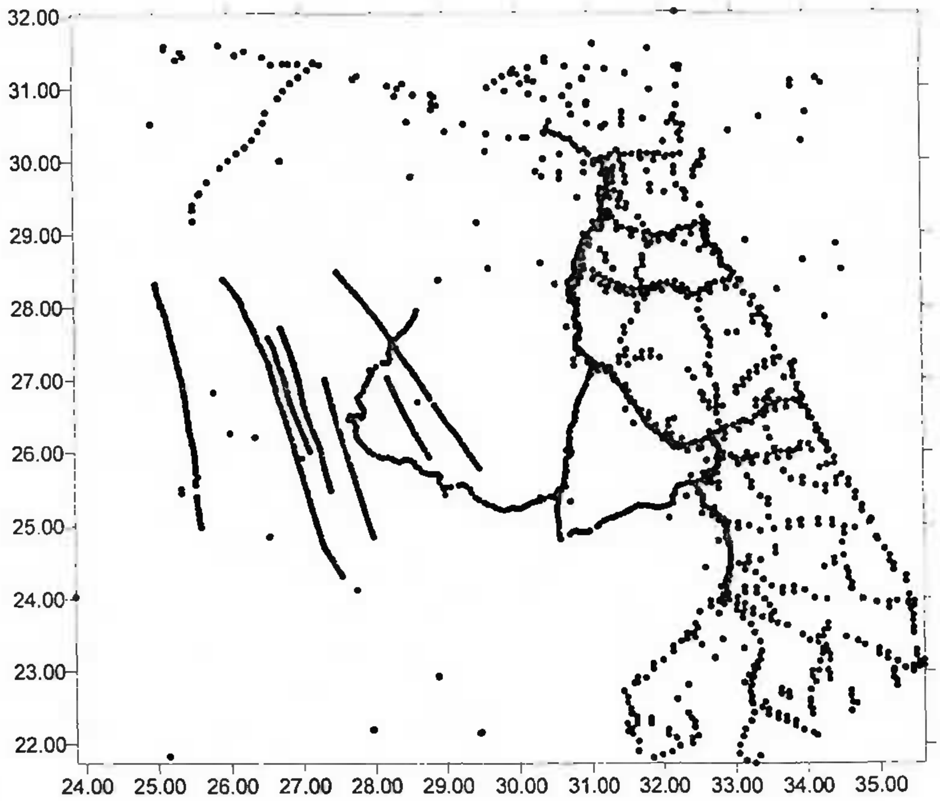

Figure 1: Available Geodetic Data in Egypt.

4. GLOBAL GEOID SOLUTIONS

Global geoid solutions represent the long wavelengths of the geoid and are obtained from global geopotential models which are given as a set of coefficients. These coefficients are those of a series of spherical harmonic functions, which are usually determined using different types of data and different computation techniques. Some global geopotential models are derived from satellite observations only and are called satellite-only solutions, GEM-9, GEM-L2, GEM-T1 and GEM-T2 are examples of such models, which contain harmonics up to degree 30, 30, 36 and 50, respectively. Other global geopotential models are obtained using altimetry and surface gravimeter added to the satellite-only solutions and they usually contain more coefficients such as, GEM10B, RAPP78, OSU-81, GPM2, OSU86F and OSU91-A which contain harmonics up to degree 36, 180, 180, 200, 360 and 360, respectively. The more coefficients in a model, the more precise the model usually is, since it contains shorter wavelength information of the Earth's gravity field [Sideris, et al., 1992].

A geoid undulation value, N , is computed in spherical approximation from a set of normalized geopotential coefficients, using the following formula:

$$N_{GM} = R \sum_{n=2}^{n\,max} \sum_{m=0}^{n} (\bar{C}^{*}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin \varphi) \qquad (1)$$

where n_{max} is the maximum degree at which the coefficients are known, $\bar{C}^{*}_{nm}$ and $\bar{S}_{nm}$ are the normalized zonal coefficients of the disturbing potential, R is the mean radius of the earth, φ and λ are the geocentric latitude and longitude, and $\bar{P}_{nm}(\sin \varphi)$ are the normalized associated Legendre functions.

The OSU-81 and OSU-91A global geopotential models have been used in this study for global geoid solutions in Egypt. The OSU-81 model has been computed to degree 180 and the OSU-91A to degree 360, both models based on a combination of the geopotential coefficients derived from the analysis of the satellite motion gravity data on a regular grid obtained from terrestrial observations as well as altimeter data. The statistical parameters of geoid undulations for the derived global geoid solutions for Egypt derived from the geopotential models are given in Table (2).

Table (2): Statistical Parameters of Geoid Undulations for The Derived Geoid of Egypt from the Global Geopotential Models (GM).

GM	N min.	N max.	N mean	RMS mean
OSU-81	7.798	23.205	14.718	3.59
OSU-91A	8.679	22.661	14.919	3.212

5. LOCAL GEOID SOLUTIONS IN EGYPT

Three different sets of data were mainly used in Egypt in order to get a local geoid. First by using heterogeneous data in a Least-squares collocation technique, second by using global geopotential models combined with terrestrial gravity data and heights from DTM, and third by combining the second with geoid undulations computed from GPS and leveling data.

5.1 Least-squares Collocation Solutions

Least-squares collocation is a mathematical technique developed by Krarup (1969) and Moritz (1972). It was developed to interpolate gravity anomaly, but has been shown to have wider applications both inside and outside physical geodesy. Thus, it is a mathematical technique for determining the earth's figure and gravitational field by a combination of heterogeneous data of different kinds. In this method one not only solves for the parameters, but also predicts the so called signals which may exist at points other than the measuring points. These signal S can be given by [Moritz, 1980]:

$$S = C_{SX} \bar{C}^{-1} X \tag{2}$$

This is the fundamental formula for least squares collocation without parameters in which C_{sx} is the covariance between the predicted signal and the measurements, and C is the covariance matrix of the measurements which contains the measuring errors.

In Egypt, the geoid has been computed relative to the datum's of WGS-72, based on Gravimetric, Doppler and Astronomic data available at this time [Alnagar, 1986]. Another geoid relative to WGS-84 was computed and based on all the available geodetic data mentioned in Table 1. Least-squares collocation has been used to predict the geoid undulations and the components of the deflection of the vertical as well as their standard error [Eltokhey,1993]. The prediction has been done at 1845 grid points (15′ x 15′) covering the Egyptian territory ($22° < \varphi < 32°$ N and $25° < \lambda < 36°$ E). The geoidal undulations computed by Alnagar, 1986 have been transformed from WGS72 to WGS84, which can be assumed to coincide with the Gravimetric reference frame, by the following formula [Denker H. and G. Wenzel, 1987]:

$$N_{WGS\text{-}84} [m] = N_{WGS\text{-}72} [m] + 4.5 \sin\varphi + 6378135\, \Delta f \sin^2\varphi - 0.6 \tag{3}$$

where $\Delta f = 0.3121057 * 10^{-6}$

The statistical parameters for the derived local heterogeneous geoids are given in Table (3).

5.2 Gravimetric Geoid Solution

The used Gravimetric geoid in this research is developed by El Sagheer, in 1995, the used data in developing this Gravimetric geoid for Egypt were as follows:

- A 5′ by 5′ grid of the global gravity anomalies trend as generated from the adopted OSU-81 Earth Geopotential Model, of degree and order 180.
- A 5′ by 5′ grid of the enhanced free-air gravity anomaly field for Egypt.
- A 5′ by 5′ Digital Terrain Model (DTM) for Egypt.

The computation has been done at 1089 grid points (15′ x 15′) covering the major part of the Egyptian territory ($24° < \varphi < 32°$ N and $25° < \lambda < 33°$ E). The methodology used in developing the 1995 Gravimetric geoid depends on combining satellite, terrestrial gravity measurements and heights from DTM to produce the required geoidal model. The geoid undulation, N, at any point consists of three parts as represented in the following equation:

$$N = N(GM) + N(\Delta g) + N(h) \qquad (4)$$

where:

N(GM) represents the contribution of the adopted geopotential model as a global trend (long wavelength feature of the geoid),

N(Δg) represents the contribution of the smoothed gravity anomalies as a regional trend (medium wavelength feature of the geoid), and

N(h) represents the contribution of the topographic local irregularities of the terrain as a local residual trend (short wavelength feature of the geoid).

The statistical parameters for the derived local gravimetric geoid are given in Table (3).

5.3 Improved Gravimetric Geoid Solution

The improved geoid is obtained from a combination of the Gravimetric geoid 95 and GPS/leveling data. These data were obtained from the FINNMAP project . The area of the project lies at the Egyptian Eastern Desert, and is subtended between longitude 29° E and longitude 36 °E. A Total of 389 stations in the project area were established using the relative GPS technique, from their corresponding orthometric heights, the geoidal undulations were obtained through the following well known formula ;

$$N = h - H \qquad (5)$$

where:

N is the geoidal undulation,
h is the ellipsoidal height,
H is the orthometric height.

Depending on the available GPS geoidal undulations, in the Eastern Desert and fixing their values, the Gravimetric geoid 95 was modified to fit the GPS undulations . A surface fitting is used in this purpose which representing a slope, rotation and shift between the two geoid surfaces as follows [Heiskanen and Moritz 1967]:

$$N(GPS)=N(Grav.) + a_1 + a_2 \cos\varphi \cos\lambda + a_3 \cos\varphi \sin\lambda + a_4 \sin\varphi \qquad (6)$$

where a_1 to a_4 are constants. Using the values of the undulations at the common points, 185 point out of 389, the four constants are calculated. Then the last equation is applied directly to all the values of the Gravimetric geoid 95 to make them fit the GPS undulation data. The results of the improved Gravimetric geoid undulations are illustrated in Table 3.

Table (3) Statistical Parameters of Geoid Undulations for the Derived Local Geoid solutions in Egypt

local geoid solution	N min.	N max.	N mean	RMS
Heterogeneous 86 [Alnagar, 86]	7.47	22.32	16.47	3.3
Heterogeneous 93 [Eltokhy, 93]	13.03	35.	23.14	5.21
Gravimetric 95 [El Sagheer, 95]	16.87	31.32	32.19	3.71
Improved by GPS [Shaker, et .al]	12.35	34.22	23.47	4.47

6. COMPARISON BETWEEN DIFFERENT GEOID SOLUTIONS AND GPS DATA

The geoidal undulations computed from the different geoid solutions are compared with the corresponding GPS undulation values at the common test area. The GPS undulations were taken as a reference for comparison, the result of these comparisons are given in Table (4).

Table (4): Statistics of Geoid Undulations differences between GPS and Geoid solutions in Egypt

Geoid solution	V min.	V max.	V mean	RMS
OSU-81	8.31	17.42	11.41	1.7
OSU-91A	7.25	16.54	10.37	1.6
Heterogeneous 86 [Alnagar, 86]	3.13	13.51	6.87	1.7
Heterogeneous 93 [Eltokhy, 93]	-4.41	5.14	-.80	1.8
Gravimetric 95 [El Sagheer, 95]	-5.34	5.42	-1.44	2.5
Improved by GPS [Shaker, et .al]	-4.09	4.99	-1.28	1.7

The statistics show that the differences between the GPS undulation data and the other geoid solutions are big for (OSU-81) and (OSU-91A) and decreases respectively with Geoid 86, Geoid 93, Geoid 95 till it reaches the improved geoid, which indicates that the global geoids were suffering from lack of local data . The Improved Gravimetric Geoid , Shaker et al., agree with the GPS undulation data with differences ranges from -4.093 to 4.987 meters. From these analysis, one can safely state that the present improved gravimetric geoid is the best geoid in hand that can be obtained from the available data in Egypt. This geoid denoted by Shaker et al, 1997 and is represented in Figures (2), (3).

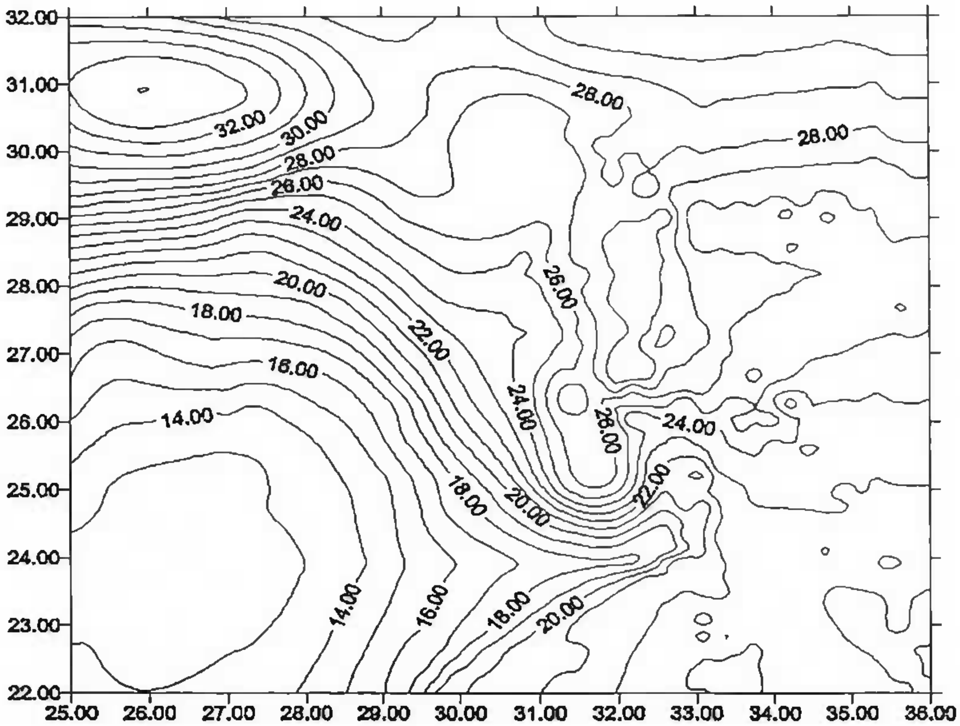

Figure (2): The Improved Egyptian Gravimetric Geoid (Shaker et. al , 97) .

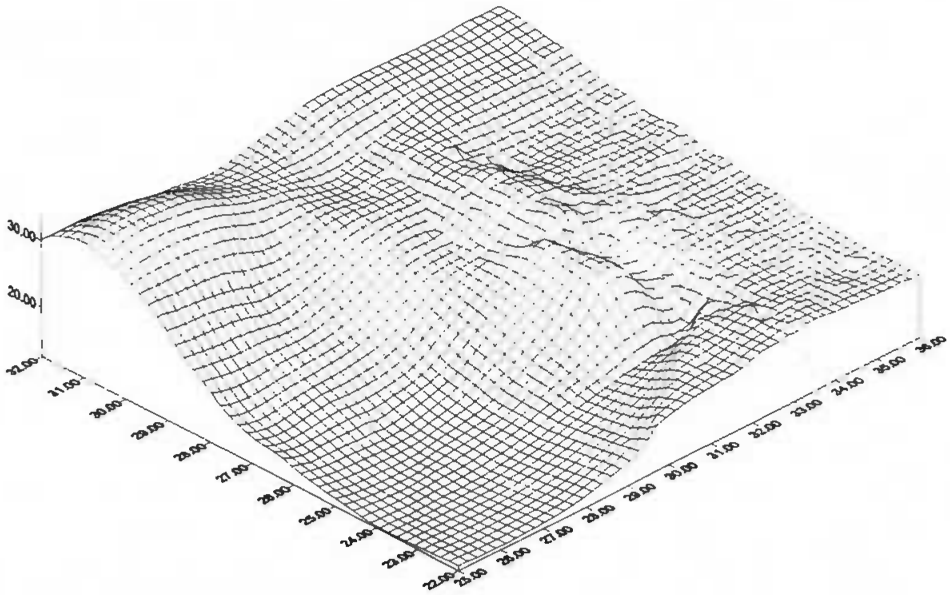

Figure (3): Three-Dimensional Geoidal Undulation Surface of Egypt.

7. CONCLUSION AND RECOMMENDATIONS

Based on the obtained results of our investigations, as outlined in the previous section, the following conclusions and recommendations are summarized as follows;

1- The currently improved gravimetric geoid conforms well with the FINNMAP-89 GPS geoid, heterogeneous geoid 1993 and the gravimetric geoid 1995, with an RMS differences in geoid undulations of about one meter.
2- Appropriate planning and more observations should be arranged to increase the quantity and quality of geodetic data in void areas.
3- It is strongly recommended to take the determined geoid solutions into consideration, for any future readjustment of the old Egyptian geodetic networks .
4- Updating the geoid information for the country using all available heterogeneous data as well as any future measurement to improve the accuracy of the obtained Egyptian geoid.
5- There should be an active and significant cooperation between the respective Egyptian surveying authorities and the responsible organizations dealing with global geopotential models computations, such that local gravity measurements, as well as any other required geodetic data, are implemented into the future refinement and enhancement of such global geopotential models. Such refined models will be more efficiently used for precise geodetic applications in Egypt, like geoid computations.

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